This is an Author's Original Manuscript (preprint) of an article submitted to 15th IFAC Conference on Programmable Devices and Embedded Systems held in Ostrava, Czech Republic, on 23–25 May 2018. The final version is available online: https://doi.org/10.1016/j.ifacol.2018.07.107.

Odometry Calibration of a Chassis with Differential Steering

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Abstract: The paper presents a method for uncertainty estimates of the position and orientation of a chassis with differential steering acquired through the odometry technique. The proposed approach was tested on a real mechanical platform. The kinematic model parameters of a chassis and coefficients needed for uncertainty computation are calibrated via reference data from an RTK GNSS receiver. This method employs an analytical expression of uncertainty propagation and can be used as an alternative to standard approaches, mainly based on Kalman or particle filters. The advantages of the presented approach include the low computation demands, deterministic calibration process, and predictable behavior.

Keywords: Mobile robot, chassis, differential steering, uncertainty estimates, odometry, position.

1. INTRODUCTION

The self-localization (obtaining the position and/or orientation) of a mobile robot embodies a fundamental task to secure the robot's autonomy. In many application, certain types of information about the accuracy of the self-localization solution are critical. The methods and algorithms (such as GNSS, INS and SLAM) utilizable for self-localization generally comprise some accuracy estimates of the provided position/orientation. More often, however, the self-localization exploits multiple data sources, and a Kalman filter is used for the data fusion and accuracy estimates can be provided via estimated variances of Kalman states. This requires proper initialization to ensure a valid accuracy estimates. The process is often not very deterministic and time-conditioned.

Another technique for providing information about the current accuracy rests in the theory of uncertainty propagation, well-known in the measurement area. The procedure requires uncertainty determination in the primary measured values and an analytical solution of uncertainty propagation to the final quantities. A related approach was analyzed and compared to Kalman uncertainty estimates by Tur (2007). Other procedures applicable to different chassis types are mentioned by Martinelli (2001) and Mirats-Tur et al. (2005).

The proposed technique was tested on an unmotorized platform with differential steering targeted for indoor map building. This platform is prepared to carry the technology described by Gabrlik et al. (2018). The projects presented by Lazna et al. (2018) and Chromy and Zalud (2014) can also integrate this technique. An accurate odometry solution with known accuracy is the data source requested for these applications. A calibrated kinematic model of the platform is needed for accurate results. In our application, the calibration process exploits as data reference a Real-Time Kinematic (RTK) solution from a GNSS receiver. The actual calibration needs to be pre-processed before the first run, after that, this step is performed only if the mechanical setup has changed. The calibration process is not fully implemented on-board the platform due to infrequent usage. The data for the calibration are measured in an outdoor environment (one offering a good open sky view to secure an accurate RTK GNSS solution), and the calibration is post-processed outside the platform.

2. METHOD

In this section, the basics of the odometry method, the uncertainty theory, and relevant application to the odometry solution are described. The technique is adjusted to 2-D movement (horizontal movement simplifies it well) in the current state of development.

2.1 Odometry of the chassis with differential steering

The standard transformation from the measured increment count to the position and orientation angle is employed. The increment count during the last sampling period with the length t_0 is

$$\Delta\Gamma_i(t) = \Gamma_i(t) - \Gamma_i(t - t_0), \quad i = 1, 2, \tag{1}$$

where $\Gamma_i(t)$ is the absolute increment count from the encoder for the wheel $\#i$ in the time t.

In general, the traveled distance s_i by the wheel $\#i$ during the t_0 period is

$$\Delta s_i = f(\Delta\Gamma_i, \Gamma_i \bmod P_i^\Gamma), \tag{2}$$

where the part $\Gamma_i \bmod P_i^\Gamma$ represents the dependence on the angle of a wheel rotation during one turn. The P_i^Γ is the increment count per one rotation of the wheel. This part can represent the variant effective wheel radius depending on the angle of a wheel rotation.

If the function f_i^w (the traveled distance per one increment depending on the increment count from a reference angle) is known, the traveled distance during the period t_0 can be computed as

$$\Delta s_i = \sum_{k=\Gamma_i(t-t_0)}^{\Gamma_i(t)} f_i^w(k \bmod P_i^\Gamma). \tag{3}$$

For the implementation, it is better to have the sum of f_i^w:

$$F_i^w(\Gamma_i) = \sum_{k=0}^{\Gamma_i} f_i^w(k \bmod P_i^\Gamma). \tag{4}$$

The traveled distance Δs_i can be computed by using the previous function as

$$\Delta s_i(t) = F_i^w(\Gamma_i(t)) - F_i^w(\Gamma_i(t-t_0)). \tag{5}$$

If the wheels are precisely centered and the radius is constant during the whole turn of a wheel, the equation (3) assumes the form

$$\Delta s_i = \Delta\Gamma_i c_i, \tag{6}$$

where c_i is the transfer constant (the units are meters per one increment).

The traveled distance by the point in the center between the wheels (a half of the track width) is

$$\Delta s = \frac{\Delta s_1 + \Delta s_2}{2}. \tag{7}$$

The change of the orientation angle (heading) is

$$\Delta\alpha = \frac{\Delta s_1 - \Delta s_2}{w}, \tag{8}$$

where w denotes track width of the drive.

The direct line length (sub-tense) from the position in time $t - t_0$ to the position in time t is

$$\Delta d = \Delta s \frac{2}{\Delta\alpha} \sin\frac{\Delta\alpha}{2} \qquad \Delta\alpha \neq 0, \tag{9}$$

$$\Delta d = \Delta s \qquad \Delta\alpha = 0. \tag{10}$$

The transformation of the position change (vector with the length $\Delta d(t)$ and angle $\alpha(t-t_0) + \Delta\alpha(t)$) into (x, y) the Cartesian coordinate system (y^+ has $\alpha = 0$) is

$$\Delta x(t) = \Delta d(t) \sin(\alpha(t-t_0) + \frac{\Delta\alpha(t)}{2}), \tag{11}$$

$$\Delta y(t) = \Delta d(t) \cos(\alpha(t-t_0) + \frac{\Delta\alpha(t)}{2}). \tag{12}$$

The position and orientation update for the time t is

$$x(t) = x(t-t_0) + \Delta x(t), \tag{13}$$

$$y(t) = y(t-t_0) + \Delta y(t), \tag{14}$$

$$\alpha(t) = \alpha(t-t_0) + \Delta\alpha(t). \tag{15}$$

The calibration of the f_1^w and f_2^w functions or c_1 and c_2 constants is possible via several criteria. Generally, it is necessary to minimize the horizontal position error of the odometry. The estimation of the transfer functions or constants is possible via minimizing the sum of errors along an estimation trajectory

$$e = \sum_{i=1}^{n} \sqrt{(x_o(i) - x_r(i))^2 + (y_o(i) - y_r(i))^2}, \tag{16}$$

where x_o and y_o are the coordinates of an odometry solution; x_r and y_r are the coordinates from the reference positioning system used as a calibration data source; i is the ID of the trajectory points acquired by the odometry and reference method. These points are sampled at the same time.

2.2 Uncertainty theory

If the indirect evaluation of the quantity y from quantities $x_1, ..., x_n$ is described by

$$y = f(x_1, x_2, ..., x_n), \tag{17}$$

the propagation of input quantities' uncertainties $\sigma_{x,1}$, ..., $\sigma_{x,n}$ to the uncertainty σ_y of the output quantity y for an indirect estimation assuming a non-existing correlation between uncertainties $\sigma_{x,1}$, ..., $\sigma_{x,n}$ is described by

$$\delta_y = \sqrt{\left(\frac{\partial f}{\partial x_1}\sigma_{x,1}\right)^2 + ... + \left(\frac{\partial f}{\partial x_n}\sigma_{x,n}\right)^2}. \tag{18}$$

More details about this topic can be found in Rabinovich (2005) or JCGM (2008).

2.3 Uncertainty of an odometry estimates

The uncertainty propagation through the equations (1) to (15) must be analyzed. The primary measured quantities are raw angles Γ_1 and Γ_2 of a wheel rotation by encoders. The situation described by (8) will be examined in this paper. The uncertainty of (1) assuming no significant correlated errors between encoders/wheels is expressible through

$$\sigma_i^{\Delta\Gamma} = \delta_i \Delta\Gamma_i, \tag{19}$$

where δ_i is an uncertainty coefficient. The final coordinates $x(t)$ and $y(t)$ are determinable by a recursive computation of previous values of x and y from $\Delta\Gamma_1$ and $\Delta\Gamma_2$. The analytical solution of this propagation is not trivial. It is possible to analyze the uncertainty in a two-step process by evaluating the uncertainties of Δx, Δy and $\Delta\alpha$, but we have to respect the fact that the uncertainties of Δx, Δy, $\Delta\alpha$, $\Delta\Gamma_1$ and $\Delta\Gamma_2$ can be strongly correlated. It is needed to have functions in these forms for an uncertainty evaluation:

$$\Delta\alpha = f^{\Delta\alpha}(\Delta\Gamma_1, \Delta\Gamma_2), \tag{20}$$

$$\Delta x = f^{\Delta x}(\Delta\Gamma_1, \Delta\Gamma_2, \alpha), \tag{21}$$

$$\Delta y = f^{\Delta y}(\Delta\Gamma_1, \Delta\Gamma_2, \alpha). \tag{22}$$

Their uncertainties are evaluated by

$$\sigma^{\Delta\alpha} = \sqrt{\left(\frac{\partial f^{\Delta\alpha}}{\partial\Delta\Gamma_1}\sigma^{\Delta\Gamma_1}\right)^2 + \left(\frac{\partial f^{\Delta\alpha}}{\partial\Delta\Gamma_2}\sigma^{\Delta\Gamma_2}\right)^2}, \tag{23}$$

$$\sigma^{\Delta x} = \sqrt{\left(\frac{\partial f^{\Delta x}}{\partial\Delta\Gamma_1}\sigma^{\Delta\Gamma_1}\right)^2 + \left(\frac{\partial f^{\Delta x}}{\partial\Delta\Gamma_2}\sigma^{\Delta\Gamma_2}\right)^2} + \left|\frac{\partial f^{\Delta x}}{\partial\alpha}\sigma^{\alpha}\right|, \tag{24}$$

$$\sigma^{\Delta y} = \sqrt{\left(\frac{\partial f^{\Delta y}}{\partial\Delta\Gamma_1}\sigma^{\Delta\Gamma_1}\right)^2 + \left(\frac{\partial f^{\Delta y}}{\partial\Delta\Gamma_2}\sigma^{\Delta\Gamma_2}\right)^2} + \left|\frac{\partial f^{\Delta y}}{\partial\alpha}\sigma^{\alpha}\right|, \tag{25}$$

The worst scenario of a strong correlation is considered in a propagation of σ^{α} to $\sigma^{\Delta x}$ and $\sigma^{\Delta y}$. The uncertainties of $\alpha(t)$, $x(t)$ and $y(t)$ can be expressed by

$$\sigma^{\alpha}(t) = \sigma^{\alpha}(t - t_0) + \sigma^{\Delta\alpha}(t), \tag{26}$$

$$\sigma^{x}(t) = \sigma^{x}(t - t_0) + \sigma^{\Delta x}(t), \tag{27}$$

$$\sigma^{y}(t) = \sigma^{y}(t - t_0) + \sigma^{\Delta y}(t). \tag{28}$$

The uncertainty coefficients δ_1 and δ_2 must be estimated during the calibration process by a statistical fitting of the predicted uncertainty to evaluated errors as the difference between the odometry and RTK GNSS solution. This should be done for a sufficient number of estimation trajectories. A proper error metric of quantities used as a minimization criterion must be considered. The most applications require a minimal horizontal position error along a trajectory of movement. This approach also indirectly minimize the heading error because the error of x, y coordinates is also affected by the heading error.

2.4 Equipment and setup

The 3-D model of the platform on which the presented approach was tested is shown in Fig. 1. The chassis is build from standard aluminum profiles and two thin precise disks with o-rings as wheels. The chassis is designed as passive (motors are not used, person manually control a motion) and it is intended as a mechanical platform for indoor scanning and map building. Wheel rotation is directly measured by precise two-channel optical incremental encoder with resolution 5,000 increments/turn per channel equipped also with 1-turn signal. The quadrature signal processing is employed thus the effective resolution is 20,000 increments/turn. The wheel diameter is approx. 0.26 m thus the motion resolution is approx. 0.04 mm/increment. The ARM Cortex-M4 microcontroller unit (MCU) STM32F401 is employed as the low level data acquisition and processing unit. The photo of the platform with all installed instrumentation needed for the calibration is shown in Fig. 2.

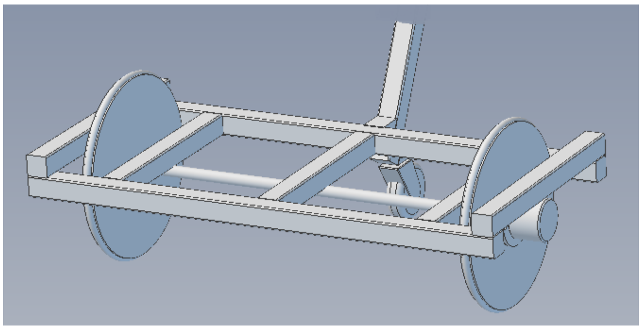

Fig. 1. 3-D model of the platform

Interconnection between components is shown in Fig. 3. Two encoders are connected to the MCU via standard quadrature signals (A, B), index signal (N) and state (failure) signal. N-signal is used to get a properly referenced angle of a wheel rotation if the calibrated function (4) or its alternative is used. A/B/N signals are differentially connected to the processing board and converted to single-ended signals due to the MCU does not support differential signals on quadrature counter module. Sampling of values from quadrature counters is time-synchronized with the GNSS receiver via pulse per second (1-PPS) signal. Absolute time tag is obtained via RS-232 interface (transmit line only). The GNSS receiver can provide measurements with 50 Hz update rate thus the encoder sampling rate is also 50 Hz in calibration mode. If the GNSS receiver is not used (indoor standard use) the sampling frequency is derived only from MCU master clock and it is not synchronized with any external accurate time source. In this mode it is possible to set a higher sampling frequency if an application requires it. An RS-232 link is used for a data output and a device configuration.

The Trimble BD982 GNSS receiver is used as the reference positioning device to evaluate the parameters of the chassis. It can provide the RTK position and orientation solution. Orientation measurement (heading + pitch/roll) requires a dual-antenna configuration. Some important parameters from manufacturer's specification (Tri (2011)) are in the table 1. The accuracy of the provided RTK position/orientation solution is sufficient for the chassis parameters estimation on trajectories several meters length.

3. RESULTS

The calibration process for the estimation of the left and right wheel diameter and the distance between wheels (track width) is done on the trajectory 70 m length.

Fig. 2. Platform with the installed instrumentation needed for the calibration of the chassis

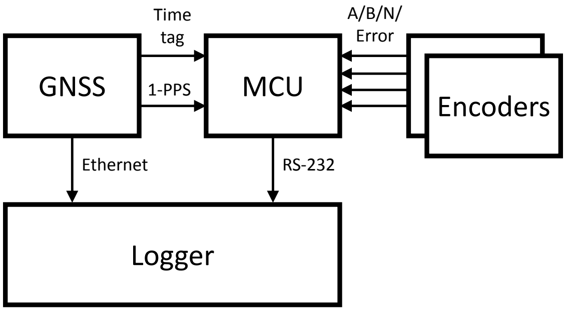

Fig. 3. Block scheme of the interconnection between the components

The trajectory used for the parameters estimation and the result of the calibrated odometry is shown in Fig. 4. The horizontal position error of the odometry using the estimated parameters on the full trajectory in Fig. 4 is shown in Fig. 5. The heading error between the odometry and the GNSS solution is evaluated in Fig. 6. The GNSS heading mesurement is possible due to a dual-antenna configuration of a rover's receiver module. The noise in the evaluated errors is caused by GNSS heading. Standard deviation of the GNSS heading measurement with a shorter base lines (about 1 m) in real applications with a dynamic motion is about 0.5 deg (based on our experiments in the past). The history of the estimated parameters on sub-trajectories 1, 2, 4 and 8 meters long (details in Fig. 9) along the estimation trajectory is shown in Fig. 7. Statistics of these obtained sets of parameters is evaluated in Fig. 8.

Table 1. Trimble BD982 specification

Parameter	Value
Position accuracy (RTK)	8/15 mm (RMS, horiz./vert.)
Heading accuracy (RTK)	0.09 deg (RMS, @2 m baseline)
Update rate	50 Hz max.
Connectivity	RS-232, CAN, USB, Ethernet
Weight	92 g
Power	2.3 W @3.3 V

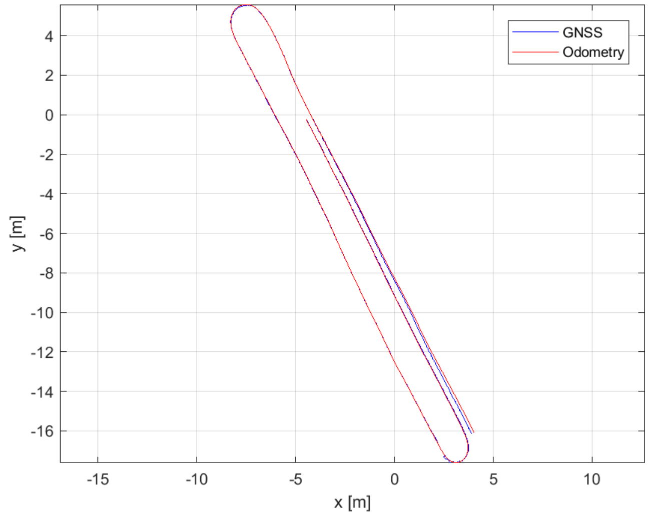

Fig. 4. Trajectory used for the estimation of the chassis parameters

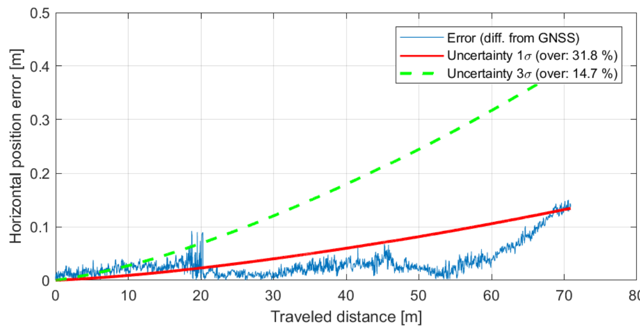

Fig. 5. Horizontal position error of the odometry along the estimation trajectory from Fig. 4

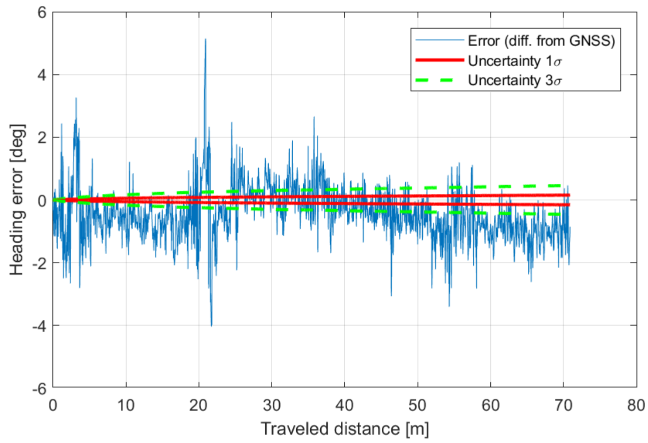

Fig. 6. Heading error of the odometry on the estimation trajectory

All estimation iterations were initiated with a very inaccurate first estimate. All parameters (designed, initial and final estimated) are in the table 2. The accuracy of the parameter estimation process is evaluated on the validation trajectory that is strongly different to the estimation trajectory. The validation trajectory has a higher heading change per one meter of a traveled distance. The trajectory is shown in Fig. 10. The distance in the horizontal plane between the RTK GNSS and odometry position is shown

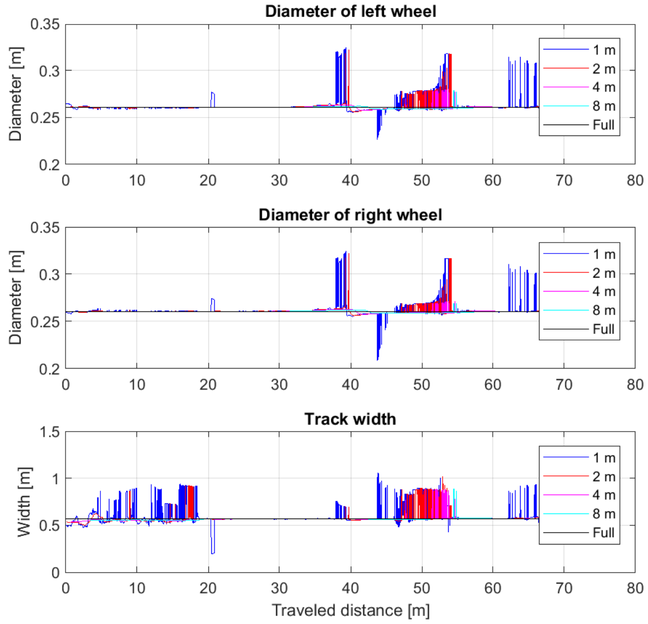

Fig. 7. Estimated chassis parameters along the trajectory according to the sub-trajectory length

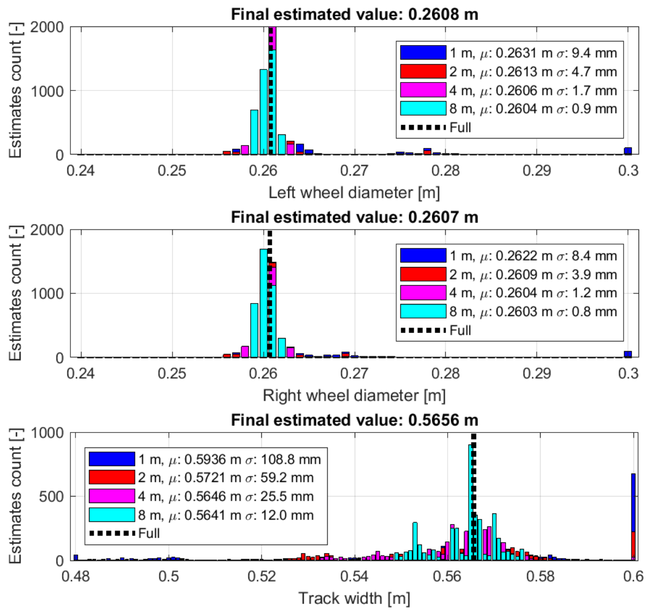

Fig. 8. Histograms of estimated chassis parameters depending on the sub-trajectory length

in Fig. 11. In this graph there are also shown 1σ and 3σ uncertainty estimates. Errors and uncertainty estimates of the chassis heading are in Fig. 12.

Table 2. Summary of the chassis parameters

Parameter	Designed	Initial est.	Final est.
Left wheel diameter	0.260 m	0.300 m	0.2608 m
Right wheel diameter	0.260 m	0.300 m	0.2607 m
Track width	0.563 m	0.700 m	0.5656 m

4. DISCUSSION

The estimated chassis parameters (table 2) are very close to the designed values and appear to be stable during

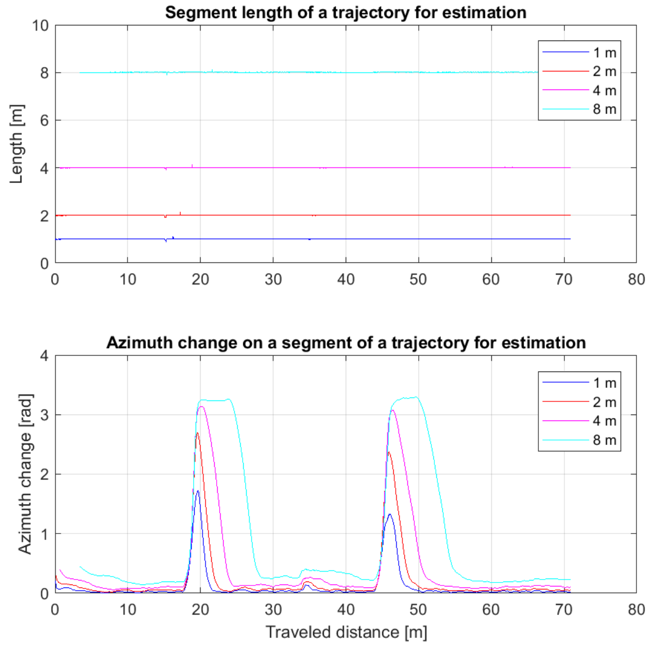

Fig. 9. Parameters of the sub-trajectories

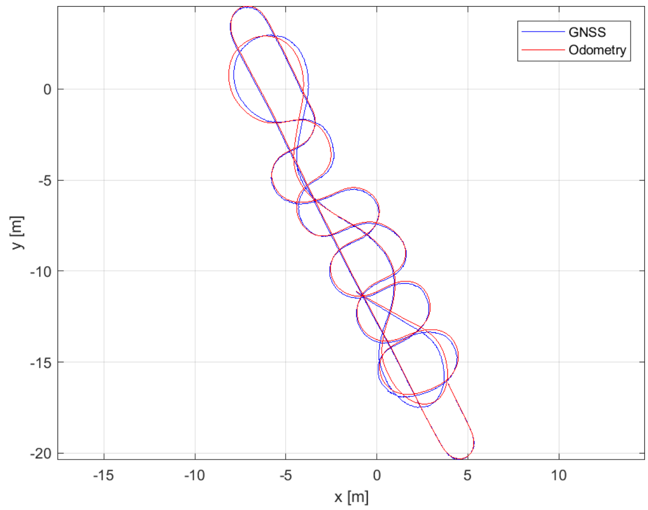

Fig. 10. Trajectory used for the validation of the estimated parameters

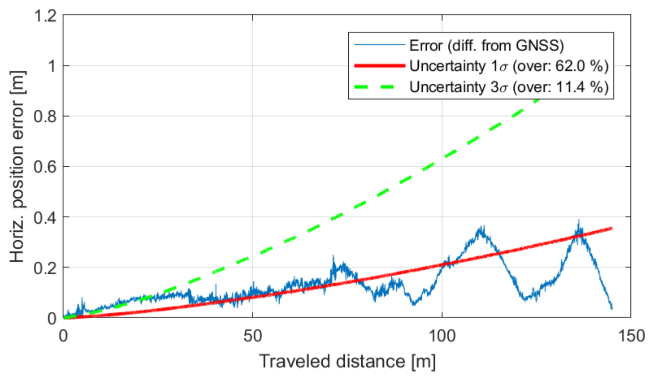

Fig. 11. Horizontal position error of the odometry on the validation trajectory

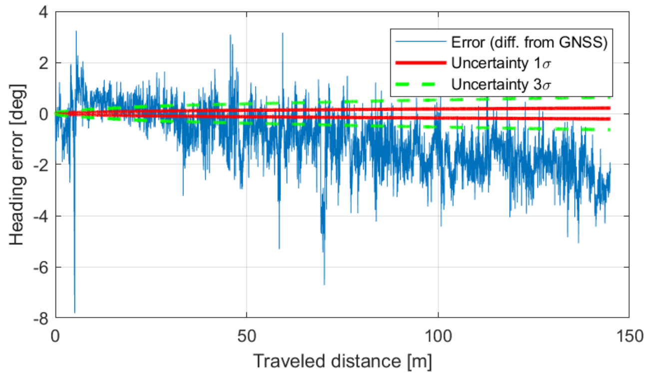

Fig. 12. Heading error of the odometry on the validation trajectory

motion (Fig. 8). In the results of the parameter estimation on partial sub-trajectories, some oscillations are noticeable (Fig. 7); these differences, however, most probably arise from inappropriate conditioning of the estimation process rather than a real change of the parameters.

Any reliable uncertainty estimates' calibration requires us to evaluate the position errors on a sufficient count of estimation trajectories. In this paper, only one estimation trajectory is used for the calibration process due to a limited amount of measured data available, but the procedure appears to run effectively under such a condition. The identified uncertainty coefficient values from table 3 approximately correspond to the uncertainty of 0.096 % from the distance traveled by a wheel. It can be also interpreted as 0.96 mm per 1 meter traveled by a wheel.

The calibration of uncertainty estimates was performed only for a horizontal position uncertainty; however, the heading uncertainty can be also obtained from the relevant processing. The reference heading measurement used in the experiment was not sufficiently accurate for conclusive assessment of the accuracy of the heading uncertainty estimates' trend, but the real error seems to rises faster than the uncertainty estimate along the estimation and the validation trajectories. Here, the cause most likely consists in the correlated errors between the encoders that exist when the chassis does not move along a straight trajectory. Moments affecting the slip or the skid act on the wheels in the opposite phase in this situation. This can be integrated into the encoder uncertainty model as a partial uncertainty based on the difference between the increment count from both encoders during the sampling period.

5. CONCLUSION

The paper presents a simple method for computing odometry uncertainty estimates, a technique applicable to different types of mobile robot chassis. Compared to advanced approaches, such as that exploiting particle filters, the procedure does not require intensive numerical computation and is thus usable for real-time calculations on a low performance microcontroller unit. The technique was verified by utilizing data measured on a chassis with differential steering. Reliable assessment of the uncertainty estimates' accuracy is feasible on a (currently unavailable) larger dataset, but the results obtained from a small dataset do not indicate serious deficiencies. The outcomes to be obtained from a larger dataset will probably refer to new dependencies that must constitute an integral part of the uncertainty propagation model.

Table 3. Summary of the estimated uncertainty coefficients

Device	Estimated parameter value
Left wheel encoder	$\delta_1 = 3.90 \cdot 10^{-8}$
Right wheel encoder	$\delta_1 = 3.90 \cdot 10^{-8}$

ACKNOWLEDGEMENTS

This work was supported by the European Regional Development Fund under the project Robotics for Industry 4.0 (reg. no. CZ.02.1.01/0.0/0.0/15_003/0000470), and by the grant no. FEKT-S-17-4234 – "Industry 4.0 in automation and cybernetics" financially supported by the Internal science fund of Brno University of Technology.

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